A Third Look At Prolog

Outline

- Numeric computation in Prolog
- Problem space search
  - Knapsack
  - 8-queens
- Farewell to Prolog

Unevaluated Terms

- Prolog operators allow terms to be written more concisely, but are not evaluated
- These are all the same Prolog term:

```
+(1,*(2,3))
1+ *(2,3)
+(1,2*3)
(1+(2*3))
1+2*3
```

- That term does *not* unify with 7

Evaluating Expressions

```
?- X is 1+2*3.
X = 7.
```

- The predefined predicate **`is`** can be used to evaluate a term that is a numeric expression
- **`is(X,Y)`** evaluates the term **`Y`** and unifies **`X`** with the resulting atom
- It is usually used as an operator

Instantiation Is Required

```
?- Y=X+2, X=1.
Y = 1+2,
X = 1.

?- Y is X+2, X=1.
ERROR: is/2: Arguments are not sufficiently instantiated
?- X=1, Y is X+2.
X = 1,
Y = 3.
```

Evaluable Predicates

- For **`X is Y`**, the predicates that appear in **`Y`** have to be *evaluable predicates*
- This includes things like the predefined operators **`+`**, **`-`**, **`*`** and **`/`**
- There are also other predefined evaluable predicates, like **`abs(Z)`** and **`sqrt(Z)`**

Real Values And Integers

```
?- X is 1/2.
X = 0.5.

?- X is 1.0/2.0.
X = 0.5.

?- X is 2/1.
X = 2.

?- X is 2.0/1.0.
X = 2.0.
```

There are two numeric types: integer and real.

Most of the evaluable predicates are overloaded for all combinations.

Prolog is dynamically typed; the types are used at runtime to resolve the overloading.

But note that the goal **`2=2.0`** would fail.

Comparisons

- Numeric comparison operators:
  `<`, `>`, `=<`, `>=`, `=:=`, `=\=`
- To solve a numeric comparison goal, Prolog evaluates both sides and compares the results numerically
- So both sides must be fully instantiated

Comparisons

```
?- 1+2 < 1*2.
false.

?- 1<2.
true.

?- 1+2>=1+3.
false.

?- X is 1-3, Y is 0-2, X =:= Y.
X = -2,
Y = -2.
```

Equalities In Prolog

- We have used three different but related equality operators:
  - `X is Y` evaluates `Y` and unifies the result with `X`: `3 is 1+2` succeeds, but `1+2 is 3` fails
  - `X = Y` unifies `X` and `Y`, with no evaluation: both `3 = 1+2` and `1+2 = 3` fail
  - `X =:= Y` evaluates both and compares: both `3 =:= 1+2` and `1+2 =:= 3` succeed (and so does `1 =:= 1.0`)
- Any evaluated term must be fully instantiated

Example: **mylength**

```
mylength([],0).
mylength([_|Tail], Len) :-
  mylength(Tail, TailLen),
  Len is TailLen + 1.
```

```
?- mylength([a,b,c],X).
X = 3.

?- mylength(X,3).
X = [_G266, _G269, _G272] .
```

Counterexample: **`mylength`**

```
mylength([],0).
mylength([_|Tail], Len) :-
  mylength(Tail, TailLen),
  Len = TailLen + 1.
```

```
?- mylength([1,2,3,4,5],X).
X = 0+1+1+1+1+1.
```

Example: **`sum`**

```
sum([],0).
sum([Head|Tail],X) :-
  sum(Tail,TailSum),
  X is Head + TailSum.
```

```
?- sum([1,2,3],X).
X = 6.

?- sum([1,2.5,3],X).
X = 6.5.
```

Example: **`gcd`**

```
gcd(X,Y,Z) :-
  X =:= Y,
  Z is X.
gcd(X,Y,Denom) :-
  X < Y,
  NewY is Y - X,
  gcd(X,NewY,Denom).
gcd(X,Y,Denom) :-
  X > Y,
  NewX is X - Y,
  gcd(NewX,Y,Denom).
```

Note: not just

`gcd(X,X,X)`

The `gcd` Predicate At Work

```
?- gcd(5,5,X).
X = 5 .


?- gcd(12,21,X).
X = 3 .

?- gcd(91,105,X).
X = 7 .

?- gcd(91,X,7).
ERROR: Arguments are not sufficiently instantiated
```

Cutting Wasted Backtracking

```
gcd(X,Y,Z) :-
  X =:= Y,
  Z is X,
  !.
gcd(X,Y,Denom) :-
  X < Y,
  NewY is Y - X,
  gcd(X,NewY,Denom),
  !.
gcd(X,Y,Denom) :-
  X > Y,
  NewX is X - Y,
  gcd(NewX,Y,Denom).
```

If this rule succeeds, there's no point in trying the others

Same here.

With those cuts, this test is unnecessary (but we'll leave it there).

Example: **fact**

```
fact(X,1) :-
  X =:= 1,
  !.
fact(X,Fact) :-
  X > 1,
  NewX is X - 1,
  fact(NewX,NF),
  Fact is X * NF.
```

```
?- fact(5,X).
X = 120.

?- fact(20,X).
X = 2432902008176640000.

?- fact(-2,X).
false.
```

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Problem Space Search

- Prolog's strength is (obviously) not numeric computation
- The kinds of problems it does best on are those that involve problem space search
  - You give a logical definition of the solution
  - Then let Prolog find it

The Knapsack Problem

- You are packing for a camping trip
- Your pantry contains these items:

Item	Weight in kilograms	Calories
bread	4	9200
pasta	2	4600
peanut butter	1	6700
baby food	3	6900

- Your knapsack holds 4 kg.
- What choice <= 4 kg. maximizes calories?

Greedy Methods Do Not Work

Item	Weight in kilograms	Calories
bread	4	9200
pasta	2	4600
peanut butter	1	6700
baby food	3	6900

- Most calories first: bread only, 9200
- Lightest first: peanut butter + pasta, 11300
- (Best choice: peanut butter + baby food, 13600)

Search

- No algorithm for this problem is known that
  - Always gives the best answer, and
  - Takes less than exponential time
- So brute-force search is nothing to be ashamed of here
- That's good, since search is something Prolog does really well

Representation

- We will represent each food item as a term **`food(N,W,C)`**
- Pantry in our example is

```
[food(bread,4,9200),
 food(pasta,2,4500),
 food(peanutButter,1,6700),
 food(babyFood,3,6900)]
```

- Same representation for knapsack contents

```
/*
   weight(L,N) takes a list L of food terms, each
   of the form food(Name,Weight,Calories).  We
   unify N with the sum of all the Weights.
*/
weight([],0).
weight([food(_,W,_) | Rest], X) :-
  weight(Rest,RestW),
  X is W + RestW.

/*
   calories(L,N) takes a list L of food terms, each
   of the form food(Name,Weight,Calories).  We
   unify N with the sum of all the Calories.
*/
calories([],0).
calories([food(_,_,C) | Rest], X) :-
  calories(Rest,RestC),
  X is C + RestC.
```

```
/*
  subseq(X,Y) succeeds when list X is the same as
  list Y, but with zero or more elements omitted.
  This can be used with any pattern of instantiations.
*/
subseq([],[]).
subseq([Item | RestX], [Item | RestY]) :-
  subseq(RestX,RestY).
subseq(X, [_ | RestY]) :-
  subseq(X,RestY).
```

- A subsequence of a list is a copy of the list with any number of elements omitted
- (Knapsacks are subsequences of the pantry)

```
?- subseq([1,3],[1,2,3,4]).
true.

?- subseq(X,[1,2,3]).
X = [1, 2, 3] ;
X = [1, 2] ;
X = [1, 3] ;
X = [1] ;
X = [2, 3] ;
X = [2] ;
X = [3] ;
X = [] ;
false.
```

Note that **`subseq`** *can do more than just* test *whether one list is a subsequence of another; it can* generate *subsequences, which is how we will use it for the knapsack problem.*

```
/*
  knapsackDecision(Pantry,Capacity,Goal,Knapsack) takes
  a list Pantry of food terms, a positive number
  Capacity, and a positive number Goal.  We unify
  Knapsack with a subsequence of Pantry representing
  a knapsack with total calories >= goal, subject to
  the constraint that the total weight is =< Capacity.
*/
knapsackDecision(Pantry,Capacity,Goal,Knapsack) :-
  subseq(Knapsack,Pantry),
  weight(Knapsack,Weight),
  Weight =< Capacity,
  calories(Knapsack,Calories),
  Calories >= Goal.
```

```
?- knapsackDecision(
|      [food(bread,4,9200),
|       food(pasta,2,4500),
|       food(peanutButter,1,6700),
|       food(babyFood,3,6900)],
|      4,
|      10000,
|      X).
X = [food(pasta, 2, 4500),
     food(peanutButter, 1, 6700)].
```

- This decides whether there is a solution that meets the given calorie goal
- Not exactly the answer we want…

Decision And Optimization

- We solved the knapsack *decision problem*
- What we wanted to solve was the knapsack *optimization problem*
- To do that, we will use another predefined predicate: **`findall`**

The `findall` Predicate

- **`findall(X,Goal,L)`**
  - Finds all the ways of proving **`Goal`**
  - For each, applies to **`X`** the same substitution that made a provable instance of **`Goal`**
  - Unifies **`L`** with the list of all those **`X`**’s

Counting The Solutions

```
?- findall(1,subseq(_,[1,2]),L).
L = [1, 1, 1, 1].
```

- This shows there were four ways of proving **`subseq(_,[1,2])`**
- Collected a list of 1's, one for each proof

Collecting The Instances

```
?- findall(subseq(X,[1,2]),subseq(X,[1,2]),L).
L = [subseq([1, 2], [1, 2]), subseq([1], [1, 2]),
     subseq([2], [1, 2]), subseq([], [1, 2])].
```

- The first and second parameters to **`findall`** are the same
- This collects all four provable instances of the goal **`subseq(X,[1,2])`**

Collecting Particular Substitutions

```
?- findall(X,subseq(X,[1,2]),L).
L = [[1, 2], [1], [2], []].
```

- A common use of **`findall`**: the first parameter is a variable from the second
- This collects all four **`X`**'s that make the goal **`subseq(X,[1,2])`** provable

```
/*
  legalKnapsack(Pantry,Capacity,Knapsack) takes a list
  Pantry of food terms and a positive number Capacity.
  We unify Knapsack with a subsequence of Pantry whose
  total weight is =< Capacity.
*/
legalKnapsack(Pantry,Capacity,Knapsack):-
  subseq(Knapsack,Pantry),
  weight(Knapsack,W),
  W =< Capacity.
```

```
/*
  maxCalories(List,Result) takes a List of lists of
  food terms.  We unify Result with an element from the
  list that maximizes the total calories.  We use a
  helper predicate maxC that takes four paramters: the
  remaining list of lists of food terms, the best list
  of food terms seen so far, its total calories, and
  the final result.
*/
maxC([],Sofar,_,Sofar).
maxC([First | Rest],_,MC,Result) :-
  calories(First,FirstC),
  MC =< FirstC,
  maxC(Rest,First,FirstC,Result).
maxC([First | Rest],Sofar,MC,Result) :-
  calories(First,FirstC),
  MC > FirstC,
  maxC(Rest,Sofar,MC,Result).
maxCalories([First | Rest],Result) :-
  calories(First,FirstC),
  maxC(Rest,First,FirstC,Result).
```

```
/*
  knapsackOptimization(Pantry,Capacity,Knapsack) takes
  a list Pantry of food items and a positive integer
  Capacity.  We unify Knapsack with a subsequence of
  Pantry representing a knapsack of maximum total
  calories, subject to the constraint that the total
  weight is =< Capacity.
*/
knapsackOptimization(Pantry,Capacity,Knapsack) :-
  findall(K,legalKnapsack(Pantry,Capacity,K),L),
  maxCalories(L,Knapsack).
```

```
?- knapsackOptimization(
|     [food(bread,4,9200),
|      food(pasta,2,4500),
|      food(peanutButter,1,6700),
|      food(babyFood,3,6900)],
|     4,
|     Knapsack).
Knapsack = [food(peanutButter, 1, 6700),
            food(babyFood, 3, 6900)] .
```

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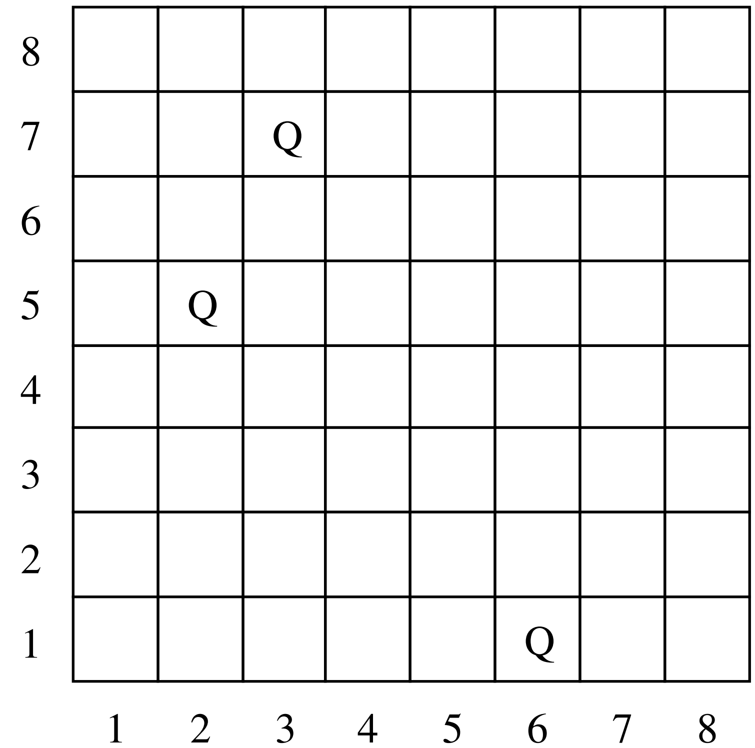
The 8-Queens Problem

- Chess background:
  - Played on an 8-by-8 grid
  - Queen can move any number of spaces vertically, horizontally or diagonally
  - Two queens are *in check* if they are in the same row, column or diagonal, so that one could move to the other’s square
- The problem: place 8 queens on an empty chess board so that no queen is in check

Representation

- We could represent a queen in column 2, row 5 with the term **`queen(2,5)`**
- But it will be more readable if we use something more compact
- Since there will be no other pieces—no **`pawn(X,Y)`** or **`king(X,Y)`**—we will just use a term of the form **`X/Y`**
- (We won't evaluate it as a quotient)

Example



- A chessboard configuration is just a list of queens
- This one is **`[2/5,3/7,6/1]`**

```
/*
  nocheck(X/Y,L) takes a queen X/Y and a list
  of queens.  We succeed if and only if the X/Y
  queen holds none of the others in check.
*/
nocheck(_, []).
nocheck(X/Y, [X1/Y1 | Rest]) :-
  X =\= X1,
  Y =\= Y1,
  abs(Y1-Y) =\= abs(X1-X),
  nocheck(X/Y, Rest).
```

```
/*
  legal(L) succeeds if L is a legal placement of
  queens: all coordinates in range and no queen
  in check.
*/
legal([]).
legal([X/Y | Rest]) :-
  legal(Rest),
  member(X,[1,2,3,4,5,6,7,8]),
  member(Y,[1,2,3,4,5,6,7,8]),
  nocheck(X/Y, Rest).
```

Adequate

- This is already enough to solve the problem: the query **`legal(X)`** will find all legal configurations:

```
?- legal(X).
X = [] ;
X = [1/1] ;
X = [1/2] ;
X = [1/3] ;
etc.
```

8-Queens Solution

- Of course that will take too long: it finds all 64 legal 1-queens solutions, then starts on the 2-queens solutions, and so on
- To make it concentrate right away on 8-queens, we can give a different query:

```
?- X = [_,_,_,_,_,_,_,_], legal(X).
X = [8/4, 7/2, 6/7, 5/3, 4/6, 3/8, 2/5, 1/1] .
```

Example

8			Q					
7						Q		
6				Q				
5		Q						
4								Q
3					Q			
2							Q	
1	Q							
	1	2	3	4	5	6	7	8

- Our 8-queens solution
- **`[8/4, 7/2, 6/7, 5/3, 4/6, 3/8, 2/5, 1/1]`**

Room For Improvement

- Slow
- Finds trivial permutations after the first:

```
?- X = [_,_,_,_,_,_,_,_], legal(X).
X = [8/4, 7/2, 6/7, 5/3, 4/6, 3/8, 2/5, 1/1] ;
X = [7/2, 8/4, 6/7, 5/3, 4/6, 3/8, 2/5, 1/1] ;
X = [8/4, 6/7, 7/2, 5/3, 4/6, 3/8, 2/5, 1/1] ;
X = [6/7, 8/4, 7/2, 5/3, 4/6, 3/8, 2/5, 1/1] ;
etc.
```

An Improvement

- Clearly every solution has 1 queen in each column
- So every solution can be written in a fixed order, like this:

```
X=[1/_,2/_,3/_,4/_,5/_,6/_,7/_,8/_]
```

- Starting with a goal term of that form will restrict the search (speeding it up) and avoid those trivial permutations

```
/*
  eightqueens(X) succeeds if X is a legal
  placement of eight queens, listed in order
  of their X coordinates.
*/
eightqueens(X) :-
  X = [1/_,2/_,3/_,4/_,5/_,6/_,7/_,8/_],
  legal(X).
```

```
nocheck(_, []).
nocheck(X/Y, [X1/Y1 | Rest]) :-
  % X =\= X1, assume the X's are distinct
  Y =\= Y1,
  abs(Y1-Y) =\= abs(X1-X),
  nocheck(X/Y, Rest).

legal([]).
legal([X/Y | Rest]) :-
  legal(Rest),
  % member(X,[1,2,3,4,5,6,7,8]), assume X in range
  member(Y,[1,2,3,4,5,6,7,8]),
  nocheck(X/Y, Rest).
```

- Since all X-coordinates are already known to be in range and distinct, these can be optimized a little

Improved 8-Queens Solution

- Now much faster
- Does not bother with permutations

```
?- eightqueens(X).
X = [1/4, 2/2, 3/7, 4/3, 5/6, 6/8, 7/5, 8/1] ;
X = [1/5, 2/2, 3/4, 4/7, 5/3, 6/8, 7/6, 8/1] ;
etc.
```

An Experiment

```
legal([]).
legal([X/Y | Rest]) :-
  legal(Rest),
  % member(X,[1,2,3,4,5,6,7,8]), assume X in range
  1=<Y, Y=<8, % was member(Y,[1,2,3,4,5,6,7,8]),
  nocheck(X/Y, Rest).
```

- Fails: “arguments not sufficiently instantiated”
- The member condition does not just *test* in-range coordinates; it *generates* them

Another Experiment

```
legal([]).
legal([X/Y | Rest]) :-
  % member(X,[1,2,3,4,5,6,7,8]), assume X in range
  member(Y,[1,2,3,4,5,6,7,8]),
  nocheck(X/Y, Rest),
  legal(Rest). % formerly the first condition
```

- Fails: “arguments not sufficiently instantiated”
- The **`legal(Rest)`** condition must come first, because it *generates* the partial solution tested by **`nocheck`**

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Parts We Skipped

- Some control predicate shortcuts
  - `->` for if-then and if-then-else
  - `;` for a disjunction of goals
- Exception handling
  - System-generated or user-generated exceptions
  - `throw` and `catch` predicates
- The API
  - A small ISO API; most systems provide more
  - Many public Prolog libraries: network and file I/O, graphical user interfaces, etc.

A Small Language

- We did not have to skip as much of Prolog as we did of ML and Java
- Prolog is a small language
- Yet it is powerful and not easy to master
- The most important things we skipped are the *techniques* Prolog programmers use to get the most out of it