## A Second Look At ML

## Outline

## - Patterns <br> - Local variable definitions <br> - A sorting example

## Two Patterns You Already Know

- We have seen that ML functions take a single parameter:

$$
\text { fun } f n=n * n \text {; }
$$

- We have also seen how to specify functions with more than one input by using tuples: fun $f(a, b)=a^{*} b ;$
- Both $\mathbf{n}$ and ( $\mathbf{a}, \mathbf{b}$ ) are patterns. The $\mathbf{n}$ matches and binds to any argument, while ( $\mathbf{a}, \mathbf{b}$ ) matches any 2-tuple and binds $\mathbf{a}$ and $\mathbf{b}$ to its components


## Underscore As A Pattern

$$
\begin{aligned}
& \text { - fun f }=\text { = "yes"; } \\
& \text { val } f=\text { fn : 'a -> string } \\
& -\mathrm{f} 34.5 \text {; "yes" : string } \\
& \text { val it = "yes" } \\
& -\mathrm{f}[] ; \\
& \text { val it = "yes" : string } \\
& \hline
\end{aligned}
$$

- The underscore can be used as a pattern
- It matches anything, but does not bind it to a variable
- Preferred to: fun $f$ x = "yes";


## Constants As Patterns

```
- fun f 0 = "yes";
Warning: match nonexhaustive
    0 => ...
val f = fn : int -> string
- f 0;
val it = "yes" : string
```

- Any constant of an equality type can be used as a pattern
- But not:

$$
\text { fun f } 0.0=\text { "yes"; }
$$

## Non-Exhaustive Match

- In that last example, the type of $\mathbf{f}$ was int -> string, but with a "match nonexhaustive" warning
- Meaning: $\mathbf{f}$ was defined using a pattern that didn't cover all the domain type (int)
- So you may get runtime errors like this:

$$
\begin{aligned}
& \text { - f 0; } \\
& \text { val it = "yes" : string } \\
& -\mathrm{f} 1 ; \\
& \text { uncaught exception nonexhaustive match failure }
\end{aligned}
$$

## Lists Of Patterns As Patterns

$$
\begin{aligned}
& \text { - fun f }[a,-]=a ; \\
& \text { Warning: match nonexhaustive } \\
& \text { a :: _ :: nil => ... } \\
& \text { val f = fn : 'a list -> 'a } \\
& \text { - f [\#"f",\#"g"]; } \\
& \text { val it = \#"f" : char }
\end{aligned}
$$

- You can use a list of patterns as a pattern
- This example matches any list of length 2
- It treats $\mathbf{a}$ and _ as sub-patterns, binding a to the first list element


## Cons Of Patterns As A Pattern

$$
\begin{aligned}
& \text { - fun } f(x:: x s)=x ; \\
& \text { Warning: match nonexhaustive } \\
& x:: \text { xs => } \ldots \\
& \text { val } f=f n: \text { 'a list }->\text { 'a } \\
& -f[1,2,3] \text {; } \\
& \text { val it }=1 \text { : int }
\end{aligned}
$$

- You can use a cons of patterns as a pattern
- X: : XS matches any non-empty list, and binds $\mathbf{X}$ to the head and $\mathbf{X S}$ to the tail
$\square$ Parens around $\mathbf{x}$ : : Xs are for precedence


## ML Patterns So Far

- A variable is a pattern that matches anything, and binds to it
- A _ is a pattern that matches anything
- A constant (of an equality type) is a pattern that matches only that constant
- A tuple of patterns is a pattern that matches any tuple of the right size, whose contents match the sub-patterns
- A list of patterns is a pattern that matches any list of the right size, whose contents match the sub-patterns
- A cons (: :) of patterns is a pattern that matches any nonempty list whose head and tail match the sub-patterns


## Multiple Patterns for Functions

$$
\begin{aligned}
& \hline- \text { fun f } 0=\text { "zero" } \\
& =\text { l f } 1=\text { "one"; } \\
& \text { Warning: match nonexhaustive } \\
& 0 \text { => ... } \\
& \text { 1 => ... } \\
& \text { val f = fn : int -> string; } \\
& - \text { f 1; } \\
& \text { val it = "one" : string }
\end{aligned}
$$

- You can define a function by listing alternate patterns


## Syntax

<fun-def> ::= fun <fun-bodies> ;
<fun-bodies> ::= <fun-body>
| <fun-body> '|' <fun-bodies>
<fun-body> ::= <fun-name> <pattern> = <expression>

- To list alternate patterns for a function
- You must repeat the function name in each alternative


## Overlapping Patterns

$$
\begin{aligned}
& \text { - fun f } 0=\text { "zero" } \\
& =\mid \quad \text { f }=\text { "non-zero"; } \\
& \text { val f }=\text { fn : int -> string; } \\
& \text { - f 0; } \\
& \text { val it = "zero" : string } \\
& \text { - f 34; } \\
& \text { val it = "non-zero" : string }
\end{aligned}
$$

- Patterns may overlap
- ML uses the first match for a given argument


## Pattern-Matching Style

- These definitions are equivalent:

$$
\begin{aligned}
& \text { fun f } 0=\text { "zero" } \\
& \text { l f }=\text { "non-zero"; } \\
& \text { fun } f= \\
& \text { if } n=0 \text { then "zero" } \\
& \text { else "non-zero"; }
\end{aligned}
$$

- But the pattern-matching style usually preferred in ML
- It often gives shorter and more legible functions


## Pattern-Matching Example

Original (from Chapter 5):
fun fact $n=$ if $n=0$ then 1 else $n$ * $\operatorname{fact}(n-1)$;

Rewritten using patterns:
fun fact $0=1$
| fact $n=n$ * $\operatorname{fact}(n-1)$;

## Pattern-Matching Example

Original (from Chapter 5):
fun reverse $L=$
if null $L$ then nil
else reverse(tl L) @ [hd L];

Improved using patterns:
fun reverse nil = nil
| reverse (first::rest) = reverse rest @ [first];

## More Examples

This structure occurs frequently in recursive functions that operate on lists: one alternative for the base case (nil) and one alternative for the recursive case (first: : rest).

Adding up all the elements of a list:
fun $f$ nil $=0$
| f (first: :rest) = first + f rest;

Counting the true values in a list:
fun $f$ nil $=0$
| $f($ true: :rest) $=1+f$ rest
f (false::rest) = f rest;

## More Examples

Making a new list of integers in which each is one greater than in the original list:

```
fun f nil = nil
    f (first::rest) = first+1 :: f rest;
```


## A Restriction

- You can't use the same variable more than once in the same pattern
- This is not legal:
fun $\mathbf{f}(\mathbf{a}, \mathbf{a})=$... for pairs of equal elements
| $\mathbf{f}(\mathbf{a}, \mathbf{b})=\ldots$ for pairs of unequal elements
- You must use this instead:
fun $f(a, b)=$
if ( $\mathbf{a}=\mathbf{b}$ ) then ... for pairs of equal elements
else ... for pairs of unequal elements


## The polyEqual Warning

```
- fun eq (a,b) = if a=b then 1 else 0;
Warning: calling polyEqual
val eq = fn : ''a * ''a -> int
- eq (1,3);
val it = 0 : int
- eq ("abc","abc");
val it = 1 : int
```

- Warning for an equality comparison, when the runtime type cannot be resolved
- OK to ignore: this kind of equality test is inefficient, but can't always be avoided


## Patterns Everywhere

$$
\begin{array}{|l}
\hline- \text { val }(\mathbf{a}, \mathbf{b})=(\mathbf{1}, \mathbf{2 . 3}) ; \\
\text { val } a=1: \text { int } \\
\text { val } b=2.3: \text { real } \\
\text { - val } a: b=[\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5 ]} ; \\
\text { Warning: binding not exhaustive } \\
\text { a : } b=\ldots \\
\text { val } a=1: \text { int } \\
\operatorname{val} b=[2,3,4,5]: \text { int list }
\end{array}
$$

- Patterns are not just for function definition
- Here we see that you can use them in a val
- More ways to use patterns, later


## Outline

## ■ Patterns

## - Local variable definitions

## - A sort example

## Local Variable Definitions

- When you use val at the top level to define a variable, it is visible from that point forward
- There is a way to restrict the scope of definitions: the let expression
<let-exp> ::= let <definitions> in <expression> end


## Example with let

$$
\begin{aligned}
& \text { - let val } x=1 \text { val } y=2 \text { in } x+y \text { end; } \\
& \text { val it }=3 \text { : int; } \\
& \text { - x; } \\
& \text { Error: unbound variable or constructor: } x
\end{aligned}
$$

- The value of a let expression is the value of the expression in the in part
- Variables defined with val between the let and the in are visible only from the point of declaration up to the end


## Proper Indentation for let

$$
\begin{aligned}
& \text { let } \\
& \text { val } x=1 \\
& \text { val } y=2 \\
& \text { in } x+y \\
& \text { end }
\end{aligned}
$$

- For readability, use multiple lines and indent let expressions like this
- Some ML programmers put a semicolon after each val declaration in a let


## Long Expressions with let

fun days2ms days $=$
let
val hours = days * 24.0
val minutes $=$ hours * 60.0
val seconds $=$ minutes * 60.0 in
seconds * 1000.0 end;

- The let expression allows you to break up long expressions and name the pieces
- This can make code more readable


## Patterns with let

$$
\begin{aligned}
& \text { fun halve nil = (nil, nil) } \\
& \text { halve [a] = ([a], nil) } \\
& \text { halve (a::b::cs) = } \\
& \text { let } \\
& \text { val ( } \mathrm{x}, \mathrm{y} \text { ) = halve cs } \\
& \text { in } \\
& \text { (a::x, b::y) } \\
& \text { end; }
\end{aligned}
$$

$\square$ By using patterns in the declarations of a let, you can get easy "deconstruction"

- This example takes a list argument and returns a pair of lists, with half in each


## Again, Without Good Patterns

 let$$
\begin{aligned}
& \text { val halved = halve cs } \\
& \text { val } x=\# 1 \text { halved } \\
& \text { val } y=\# 2 \text { halved }
\end{aligned}
$$

in
(a::x, b::y)
end;

- In general, if you find yourself using \# to extract an element from a tuple, think twice
- Pattern matching usually gives a better solution


## halve At Work

```
- fun halve nil = (nil, nil)
= | halve [a] = ([a], nil)
= | halve (a::b::cs) =
= let
= val (x, y) = halve cs
= in
= (a::x, b::y)
= end;
val halve = fn : 'a list -> 'a list * 'a list
- halve [1];
val it = ([1],[]) : int list * int list
- halve [1,2];
val it = ([1],[2]) : int list * int list
- halve [1,2,3,4,5,6];
val it = ([1,3,5],[2,4,6]) : int list * int list
```


## Outline

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## - Local variable definitions

- A sort example


## Merge Sort

- The halve function divides a list into two nearly-equal parts
- This is the first step in a merge sort
- For practice, we will look at the rest


## Example: Merge

fun merge (nil, ys) = ys
| merge (xs, nil) $=$ xs merge (x::xs, y::ys) = if ( $x<y$ ) then $x:: m e r g e(x s, y: y s)$ else y :: merge(x::xs, ys);

- Merges two sorted lists
- Note: default type for < is int


## Merge At Work

```
- fun merge (nil, ys) = ys
= | merge (xs, nil) = xs
= | merge (x::xs, y::ys) =
= if (x < y) then x :: merge(xs, y::ys)
= else y :: merge(x::xs, ys);
val merge = fn : int list * int list -> int list
- merge ([2],[1,3]);
val it = [1,2,3] : int list
- merge ([1,3,4,7,8],[2,3,5,6,10]);
val it = [1,2,3,3,4,5,6,7,8,10] : int list
```


## Example: Merge Sort

fun mergeSort nil = nil
| mergeSort [a] = [a] mergeSort theList =
let
val (x, y) = halve theList in
merge(mergeSort $x$, mergeSort $y$ ) end;

- Merge sort of a list
- Type is int list -> int list, because of type already found for merge


## Merge Sort At Work

- fun mergeSort nil = nil
= | mergeSort [a] = [a]
= | mergeSort theList $=$
$=\quad$ let
$=\quad \operatorname{val}(x, y)=$ halve theList
$=\quad$ in
$=\quad$ merge(mergeSort $x$, mergeSort $y$ )
= end;
val mergeSort = fn : int list -> int list
- mergeSort [4,3,2,1];
val it $=[1,2,3,4]$ : int list
- mergeSort [4,2,3,1,5,3,6];
val it $=[1,2,3,3,4,5,6]$ : int list


## Nested Function Definitions

- You can define local functions, just like local variables, using a let
- You should do it for helper functions that you don't think will be useful by themselves
- We can hide halve and merge from the rest of the program this way
- Another potential advantage: inner function can refer to variables from outer one (as we will see in Chapter 12)

```
(* Sort a list of integers. *)
```

fun mergeSort nil = nil
| mergeSort [e] = [e]
mergeSort theList =
let
(* From the given list make a pair of lists
* $(x, y)$, where half the elements of the
* original are in $x$ and half are in $y .{ }^{*}$ )
fun halve nil = (nil, nil)
| halve [a] = ([a], nil)
halve (a::b::cs) =
let
val ( $x, y$ ) = halve cs
in
(a::x, b::y)
end;
continued...
(* Merge two sorted lists of integers into * a single sorted list. *)
fun merge (nil, ys) = ys
| merge (xs, nil) = xs
merge (x::xs, y::ys) =
if ( $x<y$ ) then $x:: m e r g e(x s, y: y s)$
else y :: merge(x::xs, ys);
val (x, y) = halve theList in
merge(mergeSort $x$, mergeSort $y$ ) end;

## Commenting

- Everything between (* and *) in ML is a comment
- You should (at least) comment every function definition, as in any language
- what parameters does it expect
- what function does it compute
- how does it do it (if non-obvious)
- etc.

